GRADE 7 | UNIT 3

Rational Numbers

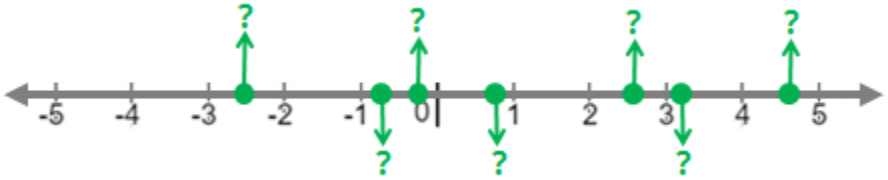
Table of Contents

GRADE 7 | MATHEMATICS

UNIT 3

Rational Numbers

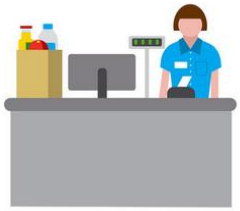
Look at the number line below. Do you notice the spaces in between the numbers? Do they also correspond to number values?



All of the values that correspond to the points on a number line are called real numbers which may be rational or irrational.

Ancient Greek mathematicians believed in the existence of rational numbers. They used them in measuring and building structures, some of which still survive to this day. Though they also were able use irrational numbers, rational numbers are more familiar to us now.

In buying and selling products using money, using terminating decimals in temperature, and cooking measurements with fractions, people use rational numbers almost every day.



In this unit, you will learn more about rational numbers, how to convert from one form to another, how to arrange them, and how to perform basic operations involving them.

Test Your Prerequisite Skills

- Performing basic operations on fractions
- Performing basic operations on decimals
- Arranging fractions in order
- Arranging decimals in order

Before you get started, answer the following items to help you assess your prior knowledge and practice some skills that you will need in studying the lessons in this unit.

1. Perform the indicated operation.
   a. $5\frac{2}{6}+\frac{2}{6}=$
   b. $4-\frac{11}{3}=$
   c. $\frac{1}{7}\times\frac{2}{5}=$
   d. $6\frac{2}{3}\div\frac{5}{4}=$

2. Perform the indicated operation.
   a. $0.1234+1.345=$
   b. $6.3251-3.01=$
   c. $2.167\times 0.75=$
   d. $10\div 0.1=$

3. Arrange the following from least to greatest.
   a. $\frac{1}{2},\ \frac{7}{2},\ \frac{9}{2},\ \frac{3}{2},\ \frac{5}{2}$
   b. $0.12,\ 0.075,\ 1.3,\ 0.175,\ 2.13$

Objectives

At the end of this unit, you should be able to

- express rational numbers from fraction form to decimal form and vice versa;
- arrange rational numbers on a number line; and
- perform operations on rational numbers.

Lesson 1: Converting Fractions to Decimals

Warm Up!

Compare and Contrast

Materials Needed: calculator, pen, paper

Instructions:

1. This activity is to be done individually.
2. A table is shown after the steps in this activity. Rewrite it on a piece of paper.
3. Take out a calculator and input the fractions indicated in the table.
4. Convert the fractions to decimals. Some natural-display calculators have a button that does this function. You may seek help from your teacher in doing this step.
5. Record these decimals in your table. If the decimals have too many decimal places, just record until the eighth decimal place.
6. Answer the following questions:
   a. Compare the decimals you got for the fractions in column A. What similarity do these decimals have?
   b. Compare the decimals you got for the fractions in column B. What similarity do these decimals have?

c. Contrast the decimals you got for the fractions in Column A from those you got for the fractions in Column B. What is their difference?

GROUP A		GROUP B	
Fraction	**Decimal**	**Fraction**	**Decimal**
$\frac{5}{2}$		$\frac{8}{3}$	
$\frac{11}{4}$		$\frac{14}{99}$	
$\frac{27}{20}$		$\frac{89}{24}$	
$\frac{137}{500}$		$\frac{63}{101}$	
$\frac{1234}{40}$		$\frac{234}{999}$	

Learn about It!

The fractions that you converted to decimals in *Warm Up!* are actually rational numbers.

Definition 1.1: **Rational numbers** are those that can be written in ratio or fraction form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

In *Warm Up!*, you have seen that when some of the fractions are converted to decimals, their decimal counterparts have either a terminating number of decimal places or have a nonterminating number of decimal places, however the digits are repeating. Thus, **terminating decimals**, as well as **repeating but nonterminating decimals**, are rational numbers too!

Converting rational numbers from fraction form to decimal form is just like dividing whole numbers using the long division algorithm which results in decimals.

Consider the following problem:

Mr. Reymundo brought with him $7\frac{11}{16}$ kg of baggage for his trip abroad. At the airport, he was informed that the maximum allowed baggage capacity is only up to 7.75 kg. Can Mr. Reymundo take all of his things with him abroad?

To help Mr. Reymundo decide if his baggage exceeds the limit, we need to compare the two values $7\frac{11}{16}$ and 7.75. In these values, the whole numbers are both, 7 so we only need to compare the fraction and decimal parts to find out the answer. We can easily compare them if they are written in the same form.

Let us then convert $\frac{11}{16}$ to decimal form as follows:

Step 1: Divide the numerator by the denominator. Align the decimal points in the dividend and the quotient.

$$16\overline{)11.}\,\overset{0.}{}$$

Step 2: Add a zero to the right of the decimal point of the dividend, and then proceed to dividing.

$$16\overline{)11.0}\,\overset{0.}{}$$

Step 3: If, after the first round of division, there is a remainder, add another zero to the right of the dividend.

$$\begin{array}{r} 0.6 \\ 16\overline{)\,11.00} \\ -96\downarrow \\ \hline 140 \end{array}$$

Step 4: Continue adding zeros until the quotient terminates. This is when no more remainder is left.

$$\begin{array}{r} 0.6875 \\ 16\overline{)\,11.0000} \\ -96 \\ \hline 140 \\ -128 \\ \hline 120 \\ -112 \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

Hence, $\frac{11}{16} = 0.6875$. Since 0.6875 is less than 0.75, Mr. Reymundo can take all of his baggage to his trip abroad.

Let's Practice!

Example 1: What is $\frac{7}{8}$ in decimal form?

Solution:

Step 1: Divide the numerator by the denominator. Align the decimal points in the dividend and the quotient.

$$8\overline{)7.}\quad \begin{matrix}0.\end{matrix}$$

Step 2: Add a zero to the right of the decimal point of the dividend, and then proceed to dividing.

$$8\overline{)7.0}\quad \begin{matrix}0.\end{matrix}$$

Step 3: If, after the first round of division, there is a remainder, add another zero to the right of the dividend.

$$\begin{array}{r} 0.8 \\ 8\overline{)7.00} \\ \underline{-6\ 4} \\ 60 \end{array}$$

Step 4: Continue adding zeros until the quotient terminates. This is when no more remainder is left.

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{64} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Therefore, $\frac{7}{8} = 0.875$.

Try It Yourself!

Write $\frac{11}{125}$ in decimal form.

Example 2: Write $2\frac{12}{25}$ in decimal form.

Solution: Since a mixed number has a whole number and a fraction part, we just convert the fraction part to a decimal and affix the whole number.

Step 1: Divide the numerator by the denominator. Align the decimal points in the dividend and the quotient.

$$25\overline{)12.}\,\overset{0.}{}$$

Step 2: Add a zero to the right of the decimal point of the dividend, and then proceed to dividing.

$$25\overline{)12.0}\,\overset{0.}{}$$

Step 3: If, after the first round of division, there is a remainder, add another zero to the right of the dividend.

$$\begin{array}{r} 0.4 \\ 25\overline{)12.00} \\ -\,100 \\ \hline 20\,0 \end{array}$$

Step 4: Continue adding zeros until the quotient terminates. This is when no more remainder is left.

$$\begin{array}{r} 0.48 \\ 25\overline{)12.00} \\ -\,100 \\ \hline 200 \\ -\,200 \\ \hline 0 \end{array}$$

Hence, $2\frac{12}{25} = 2.48$.

Try It Yourself!

What is $64\frac{13}{25}$ in decimal form?

Example 3: Write $-\frac{11}{15}$ in decimal form.

Solution:

Step 1: Divide the numerator by the denominator. Align the decimal points in the dividend and the quotient.

$$\begin{array}{r} 0. \\ 15\overline{)11.} \end{array}$$

Step 2: Add a zero to the right of the decimal point of the dividend, and then proceed to dividing.

$$\begin{array}{r} 0. \\ 15\overline{)11.0} \end{array}$$

Step 3: If, after the first round of division, there is a remainder, add another zero to the right of the dividend.

$$\begin{array}{r} 0.7 \\ 15\overline{)\,11.00} \\ -105 \\ \hline 50 \end{array}$$

Step 4: Continue adding zeros until the quotient terminates. This is when no more remainder is left.

$$\begin{array}{r} 0.7333 \\ 15\overline{)\,11.0000} \\ -105 \\ \hline 50 \\ -45 \\ \hline 50 \\ -45 \\ \hline 50 \\ -45 \\ \hline 5 \end{array}$$

Notice that the digit 3 keeps repeating, so we stop at this point.

Hence, $-\frac{11}{16} = -0.7333\ldots = -\mathbf{0.7\overline{3}}$

Note: Decimals that keep on going infinitely are called **nonterminating decimals**. **Rational numbers** can be written in the form of **repeating, nonterminating decimals**. A **bar** on top of the repeated digit or digits signifies infinite repetition.

Try It Yourself!

What is $-\frac{7}{24}$ in decimal form?

Real-World Problems

Example 4: Mrs. De Guzman owns $4\frac{7}{20}$ hectares of land which she needs to register with the Land Registration Authority (LRA). The LRA's database only accepts whole numbers and mixed decimal input. What data should Mrs. De Guzman report to the LRA?

Solution: We need to convert $4\frac{7}{20}$ to decimal.

Step 1: Divide the numerator by the denominator. Align the decimal points in the dividend and the quotient.

$$20\overline{)7.}\quad \begin{matrix}0.\end{matrix}$$

Step 2: Add a zero to the right of the decimal point of the dividend, and then proceed to dividing.

$$\begin{array}{r} 0. \\ 20\overline{)7.0} \end{array}$$

Step 3: If, after the first round of division, there is a remainder, add another zero to the right of the dividend.

$$\begin{array}{r} 0.3 \\ 20\overline{)7.00} \\ \underline{-60} \\ 100 \end{array}$$

Step 4: Continue adding zeros until the quotient terminates. This is when no more remainder is left.

$$\begin{array}{r} 0.35 \\ 20\overline{)\ 7.00} \\ -60 \\ \hline 100 \\ -100 \\ \hline 0 \end{array}$$

Therefore, $4\frac{7}{20} = 4.35$. Mrs. De Guzman should report 4.35 hectares to the LRA as the area of her piece of land.

Try It Yourself!

The ratio of the worth of Singaporean dollar to Philippine peso is 1 : 36. In Singaporean dollars, how much is ₱1 equivalent to?

Check Your Understanding!

1. Write the following in decimal form.
   a. $\frac{15}{16}$ b. $-\frac{5}{6}$ c. $11\frac{19}{20}$ d. $\frac{44}{75}$ e. $-3\frac{13}{22}$ f. $4\frac{116}{231}$

2. Kalley spent $45\frac{3}{4}$ pesos to buy a meal. How much is this amount in pesos and cents?

3. The ratio of meters to feet is approximately 1 : 3.28. How long is 1 foot in meters?

Lesson 2: Converting Decimals to Fractions

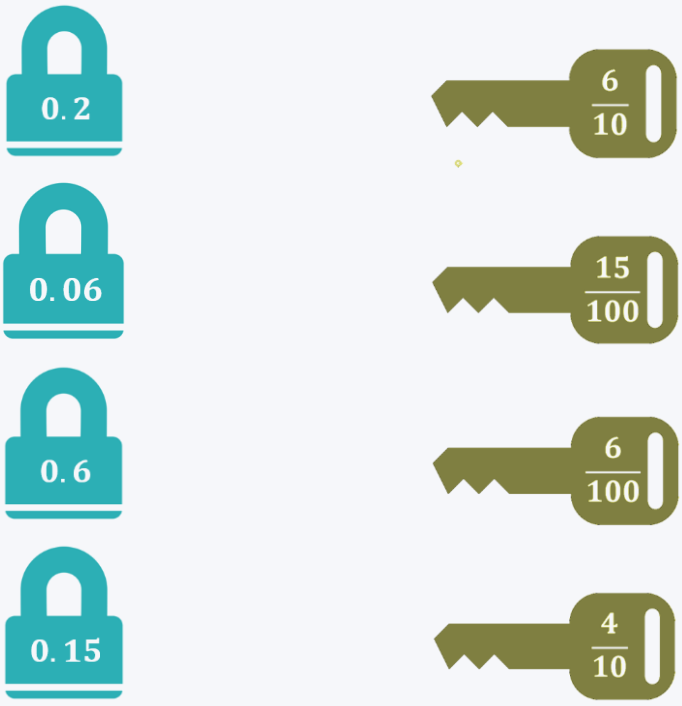
Warm Up!

Good Lock!

Materials Needed: calculator, paper, pen

Instructions:

1. This activity is to be done individually.
2. Find the key that unlocks each lock.
3. Connect each **fraction key** to its equivalent **decimal lock.**
4. You may use a calculator to check your answer.
5. Compare your answers with your seatmate.
6. List down your observations.
7. Share the patterns you see from this activity to the rest of the class.



Learn about It!

From *Warm Up!*, you may have probably noticed that decimals with one decimal place had their denominators of 10 for their equivalent fractions. You may have also noticed that those with two decimal places have denominators of 100 for their equivalent fractions. This lesson shall discuss in detail how decimals are converted to fractions manually.

Recall that the set of rational numbers only consists of fractions or terminating decimals, and repeating, nonterminating decimals. This means that only terminating decimals and repeating, nonterminating decimals can be written in fraction form.

In *Warm Up!*, the decimals given were terminating decimals. This lesson will furthermore cover the conversion of nonterminating but repeating decimals to fraction form.

Terminating Decimals

What is 0.32 in fraction form?

To convert terminating decimals to fraction form, the following steps may be followed:

Step 1: Set aside the whole number part (if any). Remove the decimal point and make the resulting number the numerator of the fraction.

$$0.32 \rightarrow 32 \rightarrow \frac{32}{?}$$

Step 2: To determine the denominator, take the place value of the last digit of the decimal. (The place value "tenths" corresponds to 10, "hundredths" to 100, "thousandths" to 1000, and so on.)

$$0.32 \rightarrow \frac{32}{100}$$

hundredths

Step 3: Express the resulting fraction in simplest form.

$$\frac{32}{100} = \frac{8}{25}$$

Repeating, Nonterminating Decimals

Converting repeating, nonterminating decimals to fraction form takes a bit more complicated process than the terminating decimals.

Consider the following problem:

Upon checking the composition of a solution, a chemist found out that it is made of $1.\overline{4}$ moles of salt (NaCl) per liter of the solution. What mixed number is equivalent to$1.\overline{4}$?

To convert $1.\overline{4}$ to a mixed number, the following steps may be followed:

Step 1: Set aside the whole number part (if any). Let n be the given decimal number.

$$n = 0.\overline{4} = 0.444\ldots$$

Step 2: Multiply *n* by a multiple of 10, depending on the number of repeating digits. If there is only one repeating digit, multiply *n* by 10. If there are two digits, multiply *n* by 100, and so on...

$$0.\overline{4}$$
1 digit
$$10n = 4.444\ldots$$

Step 3: Subtract *n* from 10*n*.

$$\begin{aligned} 10n &= 4.444\ldots \\ -\,n &= 0.444\ldots \\ \hline 9n &= 4 \end{aligned}$$

Step 4: Find the value of *n* in the resulting equation. (This must be equal to the original decimal number in Step 1.)

$$\frac{9n}{9} = \frac{4}{9} \rightarrow n = \frac{4}{9}$$

Step 5: Write the resulting fraction in its simplest form.
The resulting fraction is already in its simplest form.

Hence, $1.\overline{4} = 1\frac{4}{9}$. Therefore, the solution is made of $1\frac{4}{9}$ moles of NaCl.

Tip: You can check whether the fraction is correct by dividing the numerator by its denominator.

Let's Practice!

Example 1: Convert 3.45 to a mixed number.

Solution:

Step 1: Set aside the whole number part (if any). Remove the decimal point and make the resulting number the numerator of the fraction.

$$0.45 \rightarrow \frac{45}{?}$$

Step 2: To determine the denominator, take the place value of the last digit of the decimal. (The place value "tenths" corresponds to 10, "hundredths" to 100, "thousandths" to 1000, and so on.)

$$0.45 \rightarrow \frac{45}{100}$$

(hundredths)

Step 3: Express the resulting fraction in simplest form.

$$\frac{45}{100} = \frac{9}{20}$$

Step 4: Affix the whole number you set aside in Step 1.

$$3.45 = 3\frac{9}{20}$$

Thus, $3.45 = 3\frac{9}{20}$.

Try It Yourself!

What is the fraction form of -5.645?

Example 2: Write $0.\overline{35}$ as a fraction.

Solution:

Step 1: Set aside the whole number part (if any). Let *n* be the given decimal number.

$$n = 0.\overline{35} = 0.353535\ldots$$

Step 2: Multiply *n* by a multiple of 10, depending on the number of repeating digits. If there is only one repeating digit, multiply *n* by 10. If there are two digits, multiply *n* by 100, and so on...

$$0.\overline{35}$$
2 digits

$$100n = 35.353535\ldots$$

Step 3: Subtract *n* from 100*n*.

$$\begin{aligned} 100n &= 35.353535\ldots \\ -\ n &= \ \ 0.353535\ldots \\ \hline 99n &= 35 \end{aligned}$$

Step 4: Find the value of n in the resulting equation. (This must be equal to the original decimal number in step 1.)

$$\frac{99n}{99} = \frac{35}{99} \rightarrow n = \frac{35}{99}$$

Step 5: Write the resulting fraction in its simplest form.
The resulting fraction is already in its simplest form.

Hence, $0.\overline{35} = \frac{35}{99}$.

Try It Yourself!

Convert $8.\overline{48}$ to a fraction in lowest terms.

Example 3: How do you write $0.4\bar{2}$ in fraction form?

Solution:

Step 1: Set aside the whole number part (if any). Let n be the given decimal number.

$$n = 0.4\bar{2} = 0.4222\ldots$$

Step 2: Multiply n by a multiple of 10, depending on the number of repeating digits. If there is only one repeating digit, multiply n by 10. If there are two digits, multiply n by 100.

$$0.4\overline{2}$$
1 digit

$$10n = 4.222\ldots$$

Step 3: Subtract n from $10n$.

$$\begin{aligned} 10n &= 4.2222\ldots \\ -\ n &= 0.4222\ldots \\ \hline 9n &= 3.8 \end{aligned}$$

Step 4: Find the value of n in the resulting equation. (This must be equal to the original decimal number in step 1.)

$$\frac{9n}{9} = \frac{3.8}{9} \rightarrow n = \frac{38}{90}$$

Step 5: Write the resulting fraction in its simplest form.

$$\frac{38}{90} = \frac{19}{45}$$

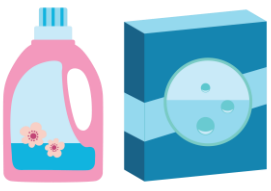
Hence, $0.4\overline{2} = \frac{19}{45}$.

Try It Yourself!

How do you write $1.5\overline{2}$ in fraction form?

Real-World Problems

Example 4: Molly discovered that the detergent she often used in laundry contains $2.3\overline{4}$ moles of calcium per kilogram. What mixed number is $2.3\overline{4}$ equal to?



Solution:

Step 1: Set aside the whole number part (if any). Let *n* be the given decimal number.

$$n = 0.3\overline{4} = 0.34444\ldots$$

Step 2: Multiply *n* by a multiple of 10, depending on the number of repeating digits. If there is only one repeating digit, multiply *n* by 10. If there are two digits, multiply *n* by 100.

$$0.3\overline{4}$$
1 digit
$$10n = 3.444\ldots$$

Step 3: Subtract *n* from 10*n*.

$$\begin{array}{r} 10n = 3.4444\ldots \\ -\ n = 0.3444\ldots \\ \hline 9n = 3.1 \end{array}$$

Step 4: Find the value of *n* in the resulting equation. (This must be equal to the original decimal number in step 1.)

$$\frac{9n}{9} = \frac{3.1}{9} \rightarrow n = \frac{31}{90}$$

Step 5: Write the resulting fraction in its simplest form.
The fraction is already in simplest form.

Step 5: Affix the whole number you set aside in step 1.

$$2.3\overline{4} = 2\frac{31}{90}$$

Hence, $2.3\overline{4} = \frac{31}{90}$.

Try It Yourself!

Upon checking the composition of a solution, a chemist found out that it is made of $1.32\overline{4}$ moles of NaCl per liter of the solution. What fraction is $1.32\overline{4}$ equivalent to?

Check Your Understanding!

1. Convert the following decimals to fractions or mixed numbers. Express your answer in simplest form.
   a. 0.45
   b. 13.596
   c. $0.\overline{18}$
   d. $-2.\overline{54}$
   e. $1.8\overline{3}$
   f. $0.8\overline{57}$

2. Daphne was feeling feverish. She measured her temperature and got a reading of 37.8°C. What is this temperature reading in mixed number form?

3. Jane used $0.\overline{6}$ meter of ribbon in wrapping a present for her niece. What is this length in fraction form?

Lesson 3: Ordering Rational Numbers

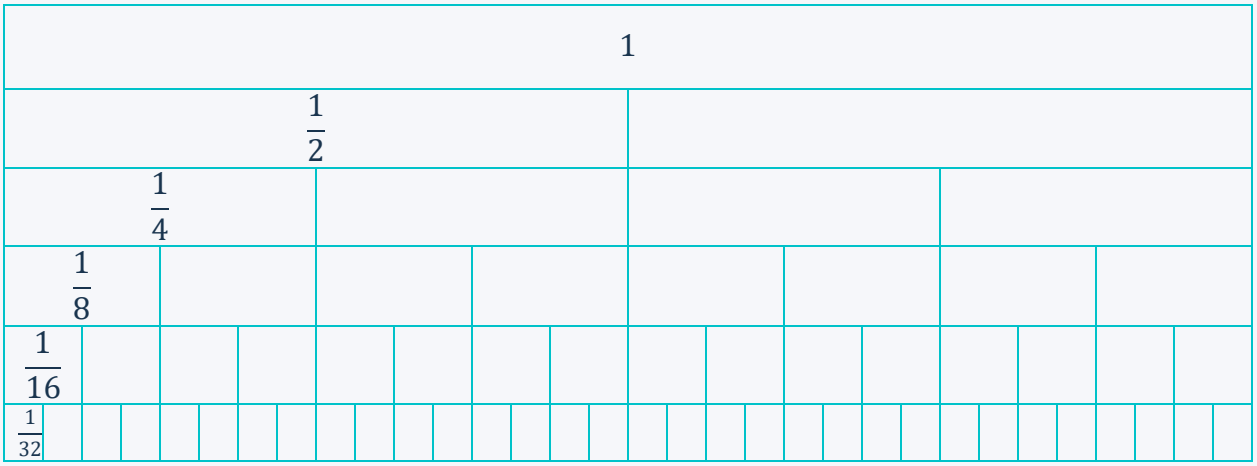
Warm Up!

Beware of Papercuts!

Materials Needed: 8 rectangular strips of paper, marker, ruler, scissors

Instructions:

1. This activity may be done in pairs.
2. Fold and cut strips of paper as shown in the figure below.



3. Model each fraction using the strips of paper then answer the following questions.
   - Is $\frac{2}{8}$ greater than $\frac{5}{16}$?
   - Is $\frac{5}{32}$ greater than $\frac{1}{4}$?
   - Is $\frac{3}{4}$ greater than $\frac{10}{16}$?
   - Is $\frac{1}{2}$ greater than $\frac{4}{32}$?
   - Is $\frac{11}{16}$ greater than $\frac{3}{8}$?

Learn about It!

In *Warm Up!,* you were able to compare rational numbers that are in fraction form using a visual means. Knowing how to compare two rational numbers is necessary in ordering them, either in ascending or descending order.

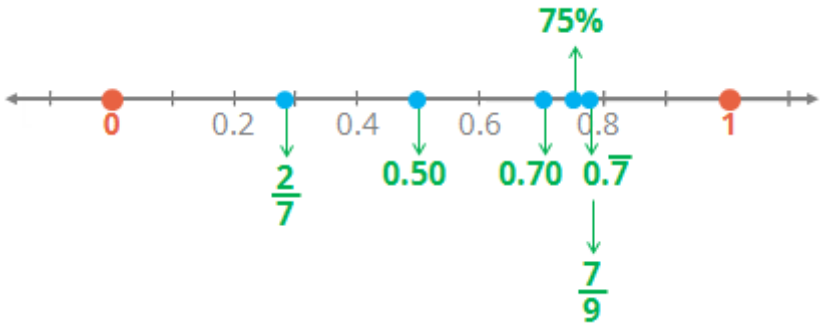
Ordering rational numbers can be done easily if the numbers are all expressed in their decimal form. For instance, we know that $0.23 < 0.5 < 1.333 \ldots$. However, ordering rational numbers in fraction form may require skills in expressing dissimilar fractions into similar ones using the concepts of prime factorization and LCD.

An even greater challenge is ordering rational numbers composed of fractions and decimals which may require conversion from one form (fraction) to another (decimal), and vice versa.

Consider the following problem:

In the 2016–2017 PBA Commissioner's Cup, the team win records are as follows: Star Hotshots, $\frac{7}{9}$; Brgy. Ginebra Kings, 75%; TNT Katropas, 0.70; NLEX Road Warriors, $\frac{2}{7}$; Alaska Aces, 0.50; and the Meralco Bolts, $0.\overline{7}$. Arrange the teams in descending win record.

Arranging the values on the number line, we see the following order:



Thus, the teams' win records arranged in descending order is as follows:

Team	Win Record
Star Hotshots	$\frac{7}{9} = 0.\overline{7} \approx 0.77$
Meralco Bolts	
Brgy. Ginebra Kings	$75\% = 0.75$
TNT Katropas	0.70
Alaska Aces	0.50
NLEX Road Warriors	$\frac{2}{7} = 0.28$

Let's Practice!

Example 1: Arrange the following numbers in increasing order:

$$\frac{3}{8}, \frac{7}{9}, \frac{5}{6}, \frac{7}{12}$$

Solution:

Step 1: Write the given as similar fractions by finding their the least common denominator (LCD). This is the same as the least common multiple (LCM) of the denominators.

$8 = 2 \cdot 2 \cdot 2$
$9 = 3 \cdot 3$
$6 = 2 \cdot 3$
$12 = 2 \cdot 2 \cdot 3$
$LCD = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

LCD = 72

Step 2: Find the equivalent fractions to the ones given using the LCD, 72.

$$\frac{3}{8}=\frac{27}{72} \quad \frac{5}{6}=\frac{60}{72}$$
$$\frac{7}{9}=\frac{56}{72} \quad \frac{7}{12}=\frac{42}{72}$$

Step 3: Arrange the fractions accordingly.

$$\frac{3}{8},\frac{7}{12},\frac{7}{9},\frac{5}{6}$$

Try It Yourself!

Arrange the following fractions in descending order:

$$\frac{7}{8},\frac{2}{5},\frac{7}{10},\frac{3}{4}$$

Example 2: Arrange the following numbers in decreasing order:

$$-2.5,\frac{7}{12},1.05,-\frac{2}{5}$$

Solution:

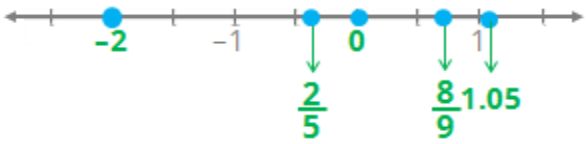
Step 1: Choose a format in which to write all the values. For this example, let us use the decimal form. This means that we have to convert the fractions $\frac{8}{9}$ and $-\frac{2}{5}$.

Step 2: Convert $\frac{8}{9}$ and $-\frac{2}{5}$ to decimals.

$$\begin{array}{r} 0.88\ldots \\ 9\overline{)8.00} \\ -\ 7\ 2 \\ \hline 80 \\ -\ 72 \\ \hline 8 \end{array} \qquad \begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ -\ 2\ 0 \\ \hline 0 \end{array}$$

So, $\frac{8}{9} = 0.\overline{8}$ and $-\frac{2}{5} = -0.4$

Step 3: Compare and arrange the values accordingly. If necessary, use a number line to guide you.



Thus, the numbers arranged from least to greatest are as follows:

$$-2, \frac{2}{5}, \frac{8}{9}, 1.05$$

Try It Yourself!

Arrange the following numbers from least to greatest:

$$4.2, -3\frac{4}{5}, 4\frac{2}{3}, -3.72$$

Example 3: Rewrite the following numbers in decimal form and arrange their decimal equivalents from least to greatest:

$$-2.1, -\frac{11}{12}, \frac{2}{5}, -\frac{10}{11}$$

Solution: The values should be written in decimal form. This means that we have to convert the fractions $-\frac{11}{12}$, $\frac{2}{5}$, and $-\frac{10}{11}$.

Step 1: Convert $-\frac{11}{12}$, $\frac{2}{5}$, and $-\frac{10}{11}$ to decimals.

$$
\begin{array}{r}
0.9166\ldots \\
12\overline{)\,11.0000} \\
-10\,8 \\
\hline
20 \\
-12 \\
\hline
80 \\
-72 \\
\hline
80
\end{array}
\qquad
\begin{array}{r}
0.4 \\
5\overline{)\,2.0} \\
-\,2\,0 \\
\hline
0
\end{array}
\qquad
\begin{array}{r}
0.9090\ldots \\
11\overline{)\,10.0000} \\
-\,9\,9 \\
\hline
10 \\
-\,0 \\
\hline
100 \\
-\,99 \\
\hline
10
\end{array}
$$

Step 2: Write the numbers as their decimal equivalents:

$$-2.1, -\frac{11}{12}, \frac{2}{5}, -\frac{10}{11} \quad \rightarrow \quad -2.1, -0.91\overline{6}, 0.4, -0.\overline{90}$$

Step 3: Compare the numbers and arrange them accordingly:

$$-2.1, -\frac{11}{12}, -\frac{10}{11}, \frac{2}{5} \quad \rightarrow \quad -2.1, -0.91\overline{6}, -0.\overline{90}, 0.4$$

Try It Yourself!

Rewrite the following numbers in decimal form and arrange their decimal equivalents from least to greatest: 3.08, $3\frac{1}{15}$, and $3\frac{2}{36}$.

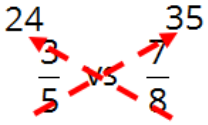
Real-World Problems

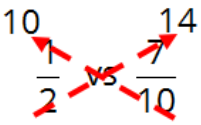
Example 4: A chemistry teacher asked her students to find the volume of the liquids in similar glass containers on their laboratory table. Then, she tasked them to create a melody of increasing pitch by lining up the beakers and hitting them with a metal rod from left to right. Based on the students' knowledge in music, this can be done by arranging the beakers from the greatest to the least volume. The students submitted he following measurements in liters: $\frac{3}{5}, \frac{7}{8}, \frac{1}{2}, \frac{7}{10}$. How should the beakers be arranged?

Tip: Rational numbers in fraction form can be compared using cross multiplication.

Solution:

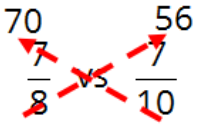
Step 1: Compare two fractions at a time. Multiply the numerator of the fraction on the left and the denominator on the right, and vice versa. Write the products on top of the numerator used.

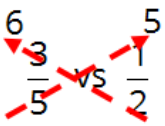




The fraction that corresponds to the higher product is bigger than the other.

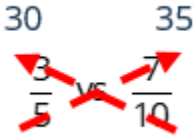
Step 2: Compare the two bigger fractions and the two smaller fractions.





We can then conclude that the biggest among the fractions is $\frac{7}{8}$ and the smallest is $\frac{1}{2}$, and the middle vaues are $\frac{3}{5}$ and $\frac{7}{10}$.

Step 3: If the middle fractions have not yet been compared, compare them using the same method to find out their order.



We can see that $\frac{7}{10}$ is bigger than $\frac{3}{5}$.

Therefore, the correct arrangement of the beakers based on their volumes is as follows:

$$\frac{7}{8}, \frac{7}{10}, \frac{3}{5}, \frac{1}{2}$$

Try It Yourself!

At a basketball tryout, the coach reviewed the information sheets submitted by the applicants. The heights of players A, B, C, D, and E in centimeters were as follows: 182.88, 156.21, $182\frac{4}{5}$, $156\frac{1}{4}$, and 182.5625.

If the coach asked the players to line up from smallest to tallest, in what order should they be?

Check Your Understanding!

1. Arrange the following sets of numbers from least to greatest:
   a. $9.3,\ 9.\overline{03},\ 9.\bar{3},\ 9.0\bar{3},\ 9.03$
   b. $\frac{1}{4},\ \frac{2}{9},\ \frac{8}{13},\ \frac{9}{16}$
   c. $\frac{8}{3},\ 2.5,\ -\frac{3}{8},\ -0.37$

2. Gelly, Holly, Ivy, Jenny, and Kelly shared a whole pizza. Gelly ate $\frac{2}{9}$ of the pizza; Holly ate $\frac{3}{16}$; Ivy ate $\frac{1}{5}$; Jenny ate $\frac{5}{27}$, and Kelly ate $\frac{4}{23}$. Who among the girls had the largest share of the pizza? Who had the least share?

3. Five kids went swimming and had a contest on who was able to hold his or her breath the longest in water. The table below shows the time they were able to stay in pool.

Name	Time (in seconds)
Elijah	25.7
Wanda	$25\frac{2}{3}$
Trinity	$25.\bar{7}$
Ron	$25\frac{3}{8}$
Greg	25.35

   Which of them was able to hold his or her breath in the longest amount of time?
   Which of them was able to hold his or her breath in the shortest amount of time?

Lesson 4: Operations on Rational Numbers

Warm Up!

Proper Modeling

Materials Needed: *cartolina*, colored paper, coloring materials, markers, scissors

Instructions:

1. This activity may be done in pairs.
2. Consider the following problems:
   - A famous pizza restaurant sells their signature pizzas in rectangular shapes. Joey and his family ordered a pizza cut into 10 equal slices. If Joey ate $\frac{1}{5}$ of $\frac{1}{2}$ of the pizza, then what part of the whole pizza did Joey eat?
   - Mary volunteers to civic duties. She makes square sandwiches for street children. She cuts each sandwich into four triangular pieces then gives exactly one piece to a child, how many children can be fed by 12 sandwiches?
3. Using the materials, create illustrations or models of the objects involved in the problem to solve for what is asked.
4. Compare your answers with the rest of the class. Did you get the same answers as the rest?

Learn about It!

From *Warm Up!,* you have actually performed operations on rational numbers that involve multiplication and division. However, you used the help of visual aids with the models and illustrations you made.

How would you perform these operations as well as addition and subtraction, for example, without the need of visual aids?

Performing operations on rational numbers is the same as adding, subtracting, multiplying, and/or dividing decimals and fractions as you learned in elementary.

Consider the following situation where operations on rational numbers are involved:

Temperature is measured in different units. The temperature in Fahrenheit (°F) is equivalent to $\frac{9}{5}$ of the temperature in degrees Celsius (°C) plus 32.

$$°\text{F} = \frac{9}{5}°\text{C} + 32$$

If the current temperature in Cebu City is 28.7°C, what is its equivalent in °F?

The fraction $\frac{9}{5}$ may be converted to decimal (1.8) before the operations are performed

$$\begin{aligned} °\text{F} &= \frac{9}{5}°\text{C} + 32 \\ &= (1.8)(28.7) + 32 \\ &= 51.66 + 32 \\ &= 83.66 \end{aligned}$$

Hence, the temperature in Cebu City is 28.7°C or 83.66°F.

Let's Practice!

Example 1: Solve for N if $N = \left(1\frac{2}{3} + \frac{5}{9}\right)\left(2\frac{5}{8} - \frac{15}{16}\right)$.

Solution: Remember to follow the correct order of operations: Solve for the values inside grouping symbols first. Next, multiply or divide, whichever comes first. Finally, add or subtract, whichever comes first.

Step 1: Find the sum and the difference of the values inside the grouping symbols. Make dissimilar fractions similar by finding the LCD, and then solving for their equivalent fractions.

$$\left(1\frac{2}{3}+\frac{5}{9}\right)=1\frac{6}{9}+\frac{5}{9}=1\frac{11}{9}=2\frac{2}{9}$$

$$\left(2\frac{5}{8}-\frac{15}{16}\right)=2\frac{10}{16}-\frac{15}{16}=1\frac{26}{16}-\frac{15}{16}=1\frac{11}{16}$$

Step 2: Perform multiplication.

$$N=\left(2\frac{2}{9}\right)\cdot\left(1\frac{11}{16}\right)$$

Convert the mixed numbers to improper fractions, and then multiply.

$$\begin{aligned}N&=\left(\frac{20}{9}\right)\cdot\left(\frac{27}{16}\right)\\&=\left(\frac{4\times5}{9}\right)\cdot\left(\frac{9\times3}{4\times4}\right)\\&=5\times\frac{3}{4}\\&=\frac{15}{4}\\&=3\frac{3}{4}\end{aligned}$$

Thus, $N=\left(1\frac{2}{3}+\frac{5}{9}\right)\left(2\frac{5}{8}-\frac{15}{16}\right)=3\frac{3}{4}$.

Try It Yourself!

Simplify: $\frac{12}{13}\left(2\frac{1}{6}+3\frac{1}{4}\right)$.

Example 2: Darla had 45 gifts to wrap as holiday giveaways to clients. She decided to use red wrappers on $\frac{1}{3}$ of the gifts, blue for $\frac{1}{5}$, and white for the remainder. How many gifts did Darla cover with white wrappers?

Solution:

Step 1: Add $\frac{1}{3}$ and $\frac{1}{5}$ to find the total fraction of the gifts that are wrapped in red or blue.

$$\frac{1}{3}+\frac{1}{5}=\frac{5}{15}+\frac{3}{15}=\frac{8}{15}$$

Step 2: Subtract the sum from 1 to find the remaining fraction of the gifts that are wrapped in white.

$$1-\frac{8}{15}=\frac{15}{15}-\frac{8}{15}=\frac{7}{15}$$

Step 3: Multiply the difference by 45 to find the number of gifts wrapped in white.

$$\frac{7}{15}\cdot\frac{45}{1}=\frac{7}{15}\cdot\frac{15\times 3}{1}=\frac{7}{1}\cdot\frac{3}{1}=21$$

Darla covered 21 gifts in white wrappers.

Try It Yourself!

How many gifts were wrapped in white wrappers in *Example 2*, if $\frac{1}{2}$ of the gifts are wrapped in red wrapper and $\frac{2}{3}$ are wrapped in blue?

Example 3: Sally has 13.5 yards of cloth. She used $\frac{1}{3}$ of the cloth to make a pillow cover and another $\frac{4}{7}$ of the cloth to make a table cloth. What fraction of the cloth in yards was left over? Express your answer in decimals.

Solution:

Step 1: Add $\frac{1}{3}$ and $\frac{4}{7}$ to find the total fraction of the cloth used by Sally.

$$\frac{1}{3}+\frac{4}{7}=\frac{7}{21}+\frac{12}{21}=\frac{19}{21}$$

Hence, $\frac{2}{21}$ of the cloth was left.

Step 2: Convert 13.5 to fraction.

$$13.5=13\frac{1}{2}=\frac{27}{2}$$

Step 3: Multiply the total length of cloth $\frac{27}{2}$ to $\frac{2}{21}$.

$$\frac{27}{2}\cdot\frac{2}{21}=\frac{3\times 9}{2}\cdot\frac{2}{3\times 7}=\frac{9}{7}=1\frac{2}{7}$$

Step 4: Convert $1\frac{2}{7}$ into decimals.

Set aside the whole number 1 and convert $\frac{2}{7}$ to decimals before affixing the whole number back again.

$$
\begin{array}{r}
0.2857142\ldots \\
7\overline{)2.0000000\ldots} \\
\underline{-14} \\
60 \\
\underline{-56} \\
40 \\
\underline{-35} \\
50 \\
\underline{-49} \\
10 \\
\underline{-7} \\
30 \\
\underline{-28} \\
20
\end{array}
$$

$1 + 0.\overline{285714} = 1.\overline{285714}$

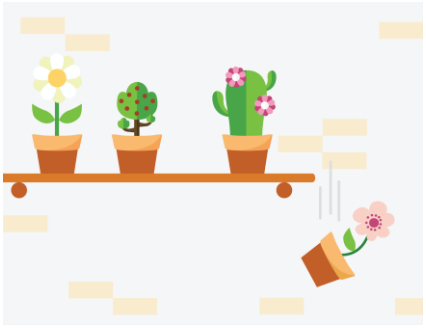
Hence, there are $1.\overline{285714}$ yards of cloth left over.

Try It Yourself!

Percy spends $\frac{1}{4}$ of his allowance on food and $\frac{3}{10}$ on transportation monthly. His mother who works abroad usually sends his allowance in dollars. If Percy's mother sent \$24.50 this month, how much would Percy spend on food if the exchange rate for peso to dollars was \$1 : ₱50.25?

More Real-World Problems

Example 4: The final velocity (v_f) of a free-falling object on Earth as it hits the ground is calculated using the formula $v_f = v_i - 9.8t$, where v_i is the initial velocity in meters per second (m/s) and t is the time in seconds (s). A flower pot was deliberately thrown from the 7th floor of a building, giving it an initial velocity of -2.3 m/s. If it took the flower pot 4.5 s to fall to the ground, what was its final velocity as it hit the ground?



Solution:

Step 1: Identify the given and substitute into the formula.

Given: $v_i = 2.3;\ t = 4.5$

$v_f = v_i - 9.8t$

$v_f = 2.3 - 9.8(4.5)$

Step 2: If needed, convert all values into one format. Let us use decimal form for this exercise.

All the given values are already in decimal format. Proceed to step 3.

Step 3: Solve. Follow the correct sequence of operations.

$$\begin{aligned} v_f &= v_i - 9.8t \\ &= -2.3 - 9.8(4.5) \\ &= -2.3 - 44.1 \\ &= -46.4 \end{aligned}$$

Hence, the final velocity of the flower pot was $\mathbf{-46.4\ m/s}$.

Try It Yourself!

Harry got the following scores in his math quizzes: $\frac{19}{20}$, 93%, $\frac{23}{25}$, and 0.89. What was his average grade in percent form?

Check Your Understanding!

1. A container is $\frac{1}{3}$ full of water and $\frac{1}{4}$ full of oil. What fraction of the container is empty?
2. Kathy spends $\frac{5}{8}$ of her money on books and another $\frac{1}{6}$ on stationaries. How much of her money was left if she initially had ₱132.50?
3. Carl used $\frac{2}{9}$ of a piece of a string to tie a parcel and $\frac{2}{5}$ of the string to tie a box. How long is left of the string if it is initially 22.345 meters long?

Challenge Yourself!

1. You knew from this unit that rational numbers may be terminating or nonterminating but repeating decimals, or fractions. Which form do you prefer more and why, decimals or fractions? Do you think nonterminating and nonrepeating decimals are rational? Why or why not?

2. Find the sum of the first 10 terms of the following sequence: $\frac{2}{5}, \frac{11}{15}, \frac{16}{15}, \ldots$

3. 1 revolution around a circle is equal to 360°. What is the measurement in degrees of the angle formed by the hands of a clock if the time shows 3:40 p.m.?

Performance Task

ABC Network Station plans on broadcasting a famous basketball All-Star game. As a sports analyst, you are tasked by the station to provide a list of the players to be included in the game. You are to research 10 suitable candidates for each the two teams.

You will need to provide a presentation and outline of each team's statistics and individual player's statistics including but not limited to the following:

- points per game
- field goal percentage
- free throw percentages

These will be shown throughout the broadcast. The data should be presented in a clear and easy-to-understand manner, highlighting the topics discussed in this unit: Rational Numbers. You should also submit the computations and sources for your information for checking and clarifications.

The presentation and layout will be approved by broadcasting head and will be submitted to the network producers. The presentation and layout will be evaluated based on accuracy, efficiency, mathematical justification, and presentation.

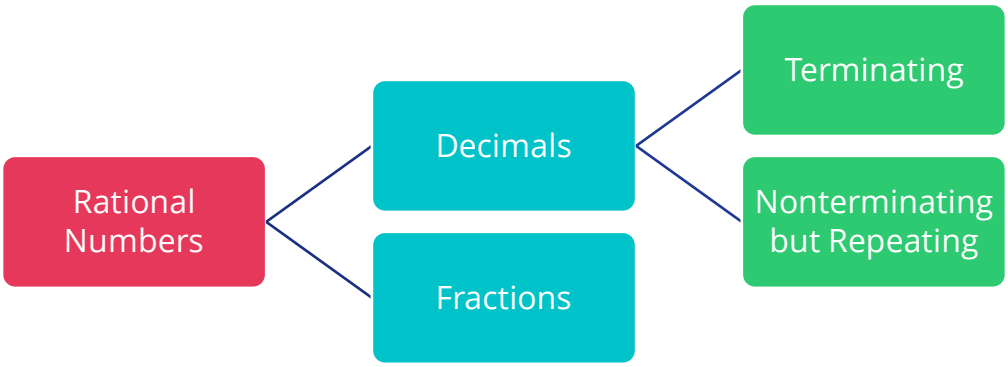
For this performance task, you may use the Internet to search any two of your favorite teams on the same league.

Performance Task Rubric

Criteria	Below Expectation (0–49%)	Needs Improvement (50–74%)	Successful Performance (75–99%)	Exemplary Performance (99+%)
Accuracy	The computations are erroneous and do not show the use of rational numbers.	The computations are erroneous and show some use of the concepts of rational numbers.	The computations are accurate and show the use of rational numbers.	The computations are accurate and show a wise use of rational numbers.
Efficiency	The design and presentation is poor.	The design and presentation is somewhat informative.	The design and presentation is informative and flawless.	The design and presentation is very informative and flawlessly done. It is also easily understandable.
Mathematical Justification	Justification is ambiguous. Only few concepts of rational numbers are applied.	Justification is not so clear. Some ideas are not connected to each other. Not all concepts of rational numbers are applied.	Justification is clear and informatively delivered. Appropriate concepts learned on rational numbers are applied	Justification is logically clear, informative, and professionally delivered. The concepts learned on rational numbers are applied and previously learned concepts are connected to the new ones.

Presentation	The overall impact of the presentation is poor.	The overall impact of the presentation is fair.	The overall impact of the presentation is impressive.	The overall impact of the presentation is highly impressive.

Wrap-up



Key Terms & Formulas

Key Terms	Descriptions
Rational Numbers	Rational numbers are numbers that can be written in ratio or fraction form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.
Fractions and Decimals	Fractions can be written in decimal form by dividing the numerator by the denominator. Decimals can be written in fraction form by using the decimal places to determine the proper denominator, and writing the fraction in lowest terms.
Ordering Rational	When arranging fractions in order, one may use the LCD and

Numbers	rewrite the fractions into their equivalents, or use cross multiplication and compare the products obtained. To arrange decimals, one only needs to compare the digits in the same place value from left to right. When fractions and decimals need to be arranged, one has to decide in which form the numbers should be written before comparing.
Performing Operations on Rational Numbers	In performing operations on rational numbers, just follow the rules for operations on fractions and decimals learned in lower year levels. When the operands are a combination of fractions and decimals, it is useful to have them all in the same form first before performing the operations. One has to be mindful to follow the correct order of operations when performing these operations.

Key to *Let's Practice!*

Lesson 1

1. 0.088
2. 64.52
3. $-0.291\overline{6}$
4. $0.02\overline{7}$

Lesson 2

1. $-5\frac{129}{200}$
2. $8\frac{16}{33}$
3. $1\frac{52}{99}$
4. $1\frac{73}{225}$

Lesson 3

1. $\frac{2}{5}, \frac{7}{10}, \frac{3}{4}, \frac{7}{8}$
2. $-3\frac{4}{5}, -3.72, 4.2, 4\frac{2}{3}$
3. $3.0\overline{6}$, $3.0\overline{76923}$, 3.08
4. $156.21, 156\frac{1}{4}, 182.5625, 182\frac{4}{5}, 182.88$

Lesson 4

1. $\frac{1}{5}$
2. 4 gifts
3. ₱307.78125
4. 92.25%

References

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Purplemath. "Adding and Subtracting Fractions." Accessed January 30, 2018. http://www.purplemath.com/modules/fraction4.htm